



Barclays Capital

Convertible Bonds: A Technical Introduction

Convertible Bonds Research – Tutorial

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Quantitative Research

Luke Olsen
+44 (20) 7773 8310
luke.olsen@barcap.com

Convertible Research

Douglas Decker, CFA
+44 (20) 7773 8302
douglas.decker@barcap.com

Haidje Rustau
+44 (20) 7773 8301
haidje.rustau@barcap.com

Judy Ho
+44 (20) 7773 9682
judy.ho@barcap.com

Convertible Bond Sales

Karam Deol
+44 (20) 7773 8320
karam.deol@barcap.com

Alan Welch
+44 (20) 7773 8320
alan.welch@barcap.com

James Bainbridge
+44 (20) 7773 8320
james.bainbridge@barcap.com

Omar Mazhar
+44 (20) 7773 8320
omar.mazhar@barcap.com

Convertible bonds have become an increasingly popular asset class in recent years, with substantial growth in markets across the globe. This has been driven by their financial benefits and risk-reward profiles, as issuers and investors alike have gained an improved understanding of these complex and fascinating products.

This report aims to solidify the reader's technical knowledge of convertibles, focusing on how the valuation of a convertible depends on its key features and characteristics.

We expect that this will appeal to a diverse cross-section of interested parties, ranging from sales/marketing professionals who wish to discuss with their clients the various opportunities presented by convertibles, to hedge fund managers, outright investors and others for whom convertibles may form an integral component of their investment strategies. It may also serve as useful supplementary material for anyone new to convertibles or who has been out of the market for some time.

For further information on convertible bonds and related products, and on the services provided by Barclays Capital, please visit our website at www.barcap.com/cbonds.

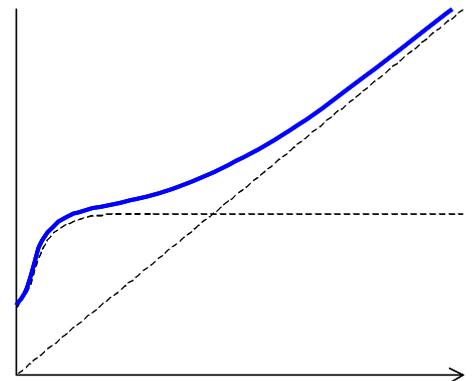
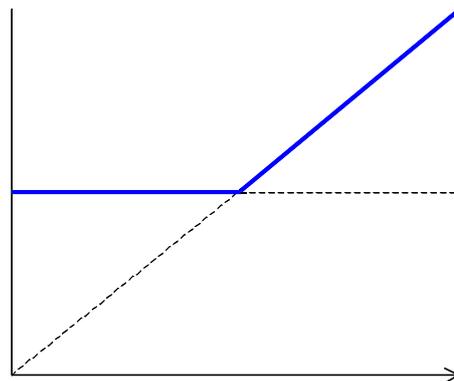


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Introduction

In this section, we highlight the key features of convertible and exchangeable bonds, noting the salient points for valuation of these securities.

A convertible or exchangeable bond is convertible into equity at the holder's option, may be called by the issuer, put by holders, and possess a variety of other features

- A convertible bond is a corporate bond that is (irrevocably) convertible at the holder's option into a specified number of equity shares, whereas an exchangeable bond is convertible into shares of a different corporate entity.
- Hence, a convertible bond may lead to the issuance of shares and dilution of the underlying equity whereas exchangeable bonds are non-dilutive because the bondholder would convert into existing shares held by the issuer of the bond.
- The holder of a convertible bond is effectively long an American call option on the underlying shares. To exercise this option, the holder surrenders the future fixed cash flows of the bond rather than paying a cash strike price.
- Exchangeable bondholders may also have an option on the issuer's credit in that converting into shares may be optimal if the issuer's credit weakens sufficiently.
- The issuer may 'call' the bond for early redemption after a certain period (the 'non-call' protection period), at a specified price (or redemption yield). Callability may also be conditional, e.g. on the underlying share price exceeding certain 'trigger' (or 'hurdle') levels for some period of time. Calls tend to restrict the holder's upside, as exercise of a call option terminates the life of the convertible and often effectively forces the holder to convert into shares.
- In some issues, the holder may 'put' the bond for early redemption to the issuer at certain future dates, at a specified price (or yield). Unlike issuer calls, holder puts tend to occur on specific dates and offer holders some downside protection.
- The bond may be in a different currency to the underlying shares, in which case the holder is also effectively long an exchange rate option (like a 'compo' option).
- Valuation and sensitivity profiles for convertible and exchangeable bonds reflect these various options and other structural features of the individual security.
- Convertibles are examples of 'hybrid' securities, with both debt and equity characteristics. This impacts accounting and capital management strategies.
- The terms of a convertible are complex and are described in detail in its offering circular ('prospectus'). This document may be hundreds of pages long, although the key descriptions of the structure are contained in the first few pages. We have over 530 prospectuses available on our website at www.barcap.com/cbonds.

Terms and Definitions

Details of terms are complex and are found in the offering circular

This section summarises the main terms of a convertible bond as outlined in an initial term sheet and described in more detail in the subsequent prospectus. Following the terms are some widely used definitions in convertible bond analysis, an example of a term sheet and a diagrammatical example of a typical convertible bond structure.

Terms

- *Bond terms*: currency; issue date, size and price; par amount; maturity date; coupon rate, frequency and day-count convention; redemption yield (yield to maturity/put/call) or redemption price, etc.
- *Conversion terms*: start and end dates (which are usually shortly after issue date and shortly before maturity, respectively); conversion ratio or number of shares per bond, (usually fixed, but often adjustable for corporate actions such as stock splits, rights issues, etc, or if there are conversion ratio resets); conversion price, which is inferred from the conversion ratio (see below).
- *Call terms*: start and end dates of the period when the issuer may redeem the bond (usually a few years post-issue through to maturity); call price – the issuer’s early redemption price (may be given by the redemption yield); trigger or ‘hurdle’ levels (usually as a percentage of the conversion price, often determined by assessing the stock price for m days out of n).
- *Put terms*: specific put dates (usually on anniversaries of the issue date); put prices – the holder’s early redemption price (may be given by the yield).
- *Contingent conversion*: holder may convert only if stock price exceeds a certain level (this feature emerged approximately a year ago in the US market and enables companies to report non-diluted earnings figures in their reports).
- *Contingent payment*: bond pays interest only if stock price and/or dividends exceed certain levels (this feature also emerged approximately a year ago in the US market and enables companies to deduct interest expense at their cost-of-debt rate).
- *Make-whole payment*: bondholder receives an additional payment equal to unpaid coupons for some initial period (typically the first few years) if the issuer calls the bonds for redemption during this period.
- *Coupon and/or dividend entitlement upon conversion*. For example, with some bonds, holders forfeit a coupon payment if they decide to convert the bond into shares on that payment date (this is known as a ‘screw clause’). An example of a dividend entitlement clause is where holders forego all dividends on ordinary shares during the financial year in which they convert the bond.
- *Cash instead of shares on conversion*: at the issuer’s option, the cash equivalent amount may be determined by an average stock price. Details, such as the calculation method for the average, are given in the prospectus.
- *Shares instead of cash on redemption*: at the issuer’s option, the number of shares deliverable in lieu of the cash redemption price is determined by an average stock price. Again, see the prospectus for details.
- *Takeover protection*: describes what happens if a relevant ‘change of control’ occurs (e.g. bondholder’s put if an acquisition results in a lower credit rating). What actually triggers a change of control may vary from one prospectus to another, but a ‘change of control’ could be defined as when more than 50% of a company’s stock changes ownership.

- *Indicative terms*: indicative ranges are supplied to potential investors for the initial conversion premium and the redemption yield from the announcement date until final pricing of the issue, when these terms become fixed.
- *Greenshoe option*: if a new issue is oversubscribed then the issuer may increase the issue size up to a certain limit (usually an extra 15%). This is intended to allow the underwriter to stabilise the price of the bonds immediately after they begin trading in the secondary market.

Definitions

- *Straight bond value, investment value or bond floor*: net present value of the fixed cash flows of the convertible bond (adjusted upwards for the possibility of early redemptions if it is optimal for the holder to exercise any put options). This is also the value of the bond portion of a convertible, or where a non-convertible bond with otherwise similar features would trade.
- *Conversion price*: par amount / conversion ratio (x exchange rate, if the underlying stock currency is different to the bond currency).
- *Parity*: stock price x conversion ratio (/ exchange rate), usually expressed as a percentage of par; parity is therefore equal to the value of the shares underlying the convertible bond.
- *Conversion premium*: bond price minus parity, usually expressed as a percentage of parity; at issue, conversion price = (1 + conversion premium) x stock price. Conversion premium can be interpreted as the extra amount an investor must pay to own the same number of shares via the convertible.

Sensitivities

- *Delta*: equity sensitivity = change in value of the convertible bond per unit change in parity (usually expressed as a percentage; e.g. a convertible with 50% delta increases in value by half a point if parity increases by one point).
- *Gamma*: equity sensitivity of delta = change in value of the convertible bond delta per unit change in parity (may be expressed as a percentage; e.g. the percentage delta of a convertible with 2% gamma and 50% delta increases to 52% if parity increases by one point).
- *Vega*: volatility sensitivity = change in value of the convertible bond per unit change in volatility (e.g. a convertible with 40% vega increases in value by 0.4 points if the equity volatility increases from 30% to 31%).
- *Rho*: interest rate sensitivity = change in value of the convertible bond per unit change in the risk-free yield curve (e.g. a convertible with 3% rho increases in value by 0.3 points if the yield curve increases in parallel from 5.0% to 5.1%); rho can also be specified as a vector, describing the sensitivities to each point in a yield curve.

Sample Termsheet

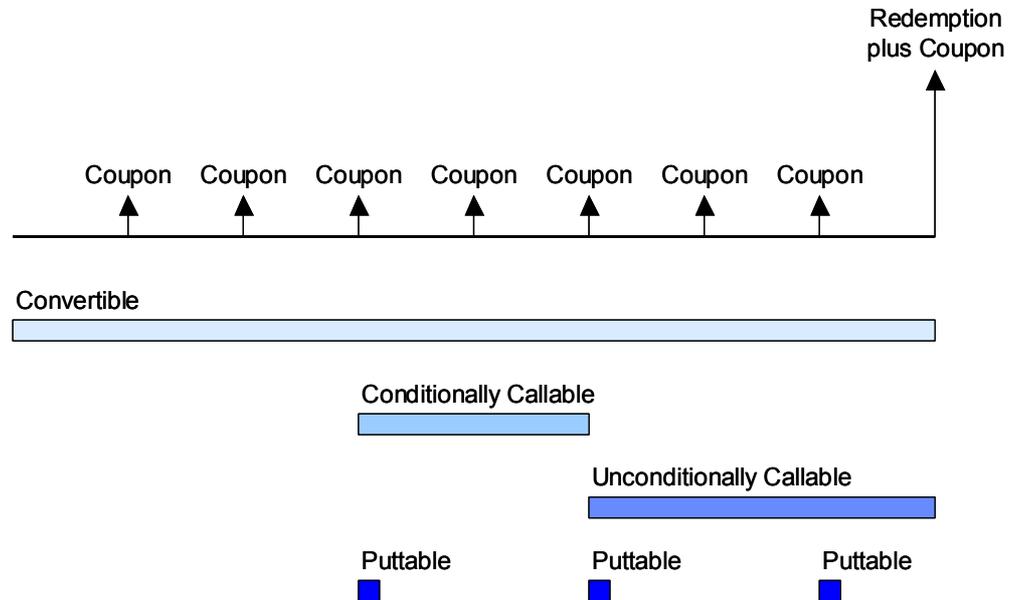
Deutsche Bank – Novartis Exchangeable Bond, November 2001, Barclays acted as co-lead manager. Please see Appendix for termsheet.

This deal was offered in two equally sized tranches of €1.4bn: one maturing in 2010 with a 3.125% redemption yield and the other maturing in 2011 with a 2.75% redemption yield. Both came to market with 28.2% initial conversion premiums. The indicated yields had been 2.625-3.125% for the 2010 issue and 2.25-2.75% for the 2011 issue. Both tranches had indicated initial conversion premiums of 28-33%.

Example Convertible Bond Structure

Figure 1 shows a diagrammatical example of an eight-year convertible bond that pays annual coupons, is convertible throughout its life, is non-callable for the first three years, becomes conditionally callable (i.e. subject to a stock price trigger) in the fourth and fifth years, then unconditionally callable for the final three years, and is puttable on the third, fifth and seventh anniversaries.

Figure 1: Time Line of an Example Convertible Bond with Calls and Puts



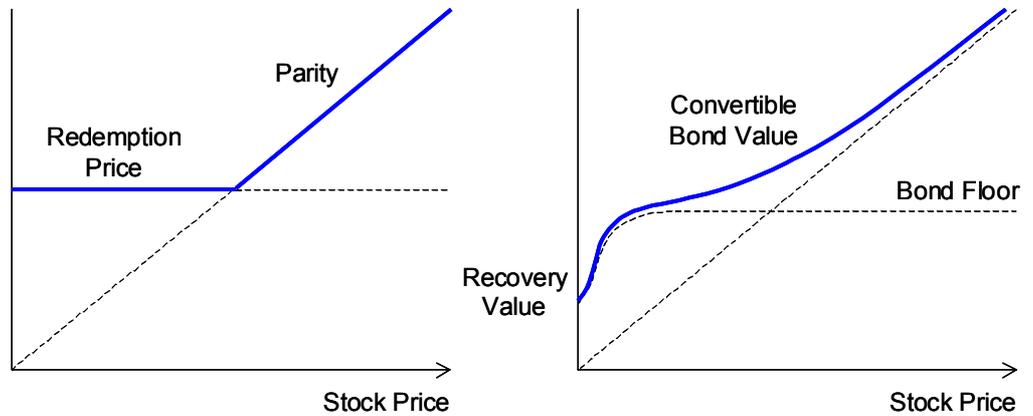
Source: Barclays Capital.

Although this is a typical structure, not all bonds are callable and puttable. Also, whilst most are callable, not all of those have both conditional and unconditional call periods.

Payoff and Valuation Profile

Here we illustrate typical profiles for the present value (or price) of a convertible bond for a range of underlying stock prices, together with its payoff at maturity, bond floor and parity (see Figure 2). These profiles are explained below.

Figure 2: Payoff and Valuation Profile of a Typical Convertible Bond



Source: Barclays Capital.

Convertible value converges to parity at high stock prices and to the bond floor at low stock prices

- Payoff at maturity is the greater of parity and the redemption price (which may be par, and may or may not include a final coupon payment).
- At any time prior to maturity, the convertible bond value is bounded below by parity and by the equivalent straight bond value, i.e. the 'bond floor'.
- The convertible bond value approaches parity as the stock price rises because, in the limit, there is no time value, i.e. no 'optionality'. In fact, it may be optimal for the investor to convert early, depending on the relative levels of the bond coupon interest, stock dividends and stock borrowing costs.
- The value of a corporate bond typically falls as the equity decreases towards zero due to the increased risk of default. In this event, bondholders compete with other creditors (on the basis of their relative ranking and subordination) to extract as much value as possible from the restructuring or liquidation: this is the 'recovery value'.

The Convertibles Market and New Issuance

Sector, Currency and Geographic Profile

- Companies in all sectors have raised financing through convertible issuance, but this was particularly appealing to growth companies such as TMTs (Telecom, Media and Technology) in recent years.
- Most new convertibles are denominated in USD, EUR, GBP, CHF or JPY.
- Companies around the globe issue convertibles. 2001, for example, saw new convertibles from companies in Greece, Korea, India and South Africa.

Issuance Data for Year 2001

Figure 3 shows data for 2001 versus 2000 (in parentheses, where available).

Figure 3: Issuance Data 2001 Vs 2000

Region	Number	Size, US\$bn	Average Yield	Average Premium
Europe	72 (47)	45.7 (28.1)	3.11% (3.89%)	26.0% (23.7%)
US	202 (146)	95.5 (61.6)	3.51%	28.7%
Asia	22 (14)	7.82 (5.26)	3.14% (2.79%)	19.3% (19.2%)

Source: Barclays Capital.

For a detailed review of new convertible issuance and the market in 2001, together with our outlook and recommendations for 2002, please refer to our research publication, *The European Convertible Market in 2002: Will the Boom Continue?* dated 16 January 2002.

Why Issue Convertibles?

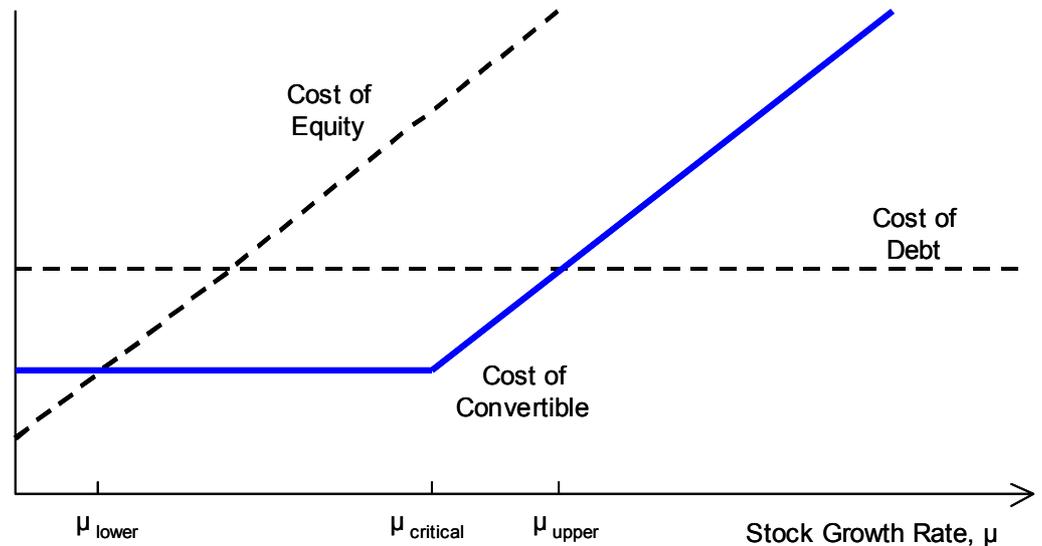
Companies choose to issue convertible/exchangeable bonds instead of straight bonds or equity, or at a certain time, for a variety of reasons, including:

- Reduced cost of capital versus straight equity or straight debt:
 - If a company issues equity then (assuming no premium/discount on sale) its cost is the dividend yield plus the stock growth rate – i.e. the opportunity cost in selling away the growth of the company.
 - If it issues straight debt then its cost is the par coupon rate, or the yield on straight debt, which is a function of the company's credit strength, balance sheet structure and access to debt capital.
 - Convertible bonds, however, enable the company to either sell (deferred) equity at a considerable premium if the holders convert or issue straight debt with a significantly reduced coupon/yield if the bonds are redeemed. Whether the bonds are converted or redeemed depends largely on the realised stock growth rate.
 - Therefore, if the stock growth rate is in the range μ_{lower} to μ_{upper} then the convertible bond offers the cheapest form of financing. Note that μ_{critical} is the stock growth rate above which holders would choose to convert into shares and below which they would redeem the bond (see Figure 4).

Convertible issuance soared in 2001, driven by investor demand and very favourable market conditions for convertible financing

Convertibles may offer a lower cost of capital to an issuer than straight debt or equity

Figure 4: Relative Costs of Straight Debt, Equity and Convertible Financing



Source: Barclays Capital.

Strong demand from investors has enabled issuers to price bonds on favourable terms

Companies can time a new convertible issue to exploit positive sentiment on the stock

Convertible issuance enables companies to tap a more diverse investor base and to raise market profile

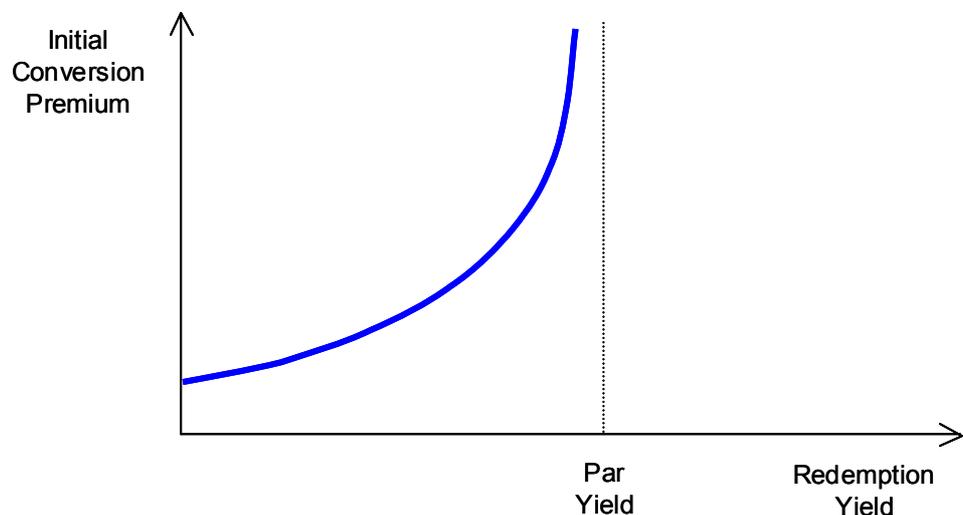
- Valuation/pricing:
 - The market forces of supply and demand – both from investors and from banks that are competing for the deals – mean that convertible bonds may be priced on very favourable terms to the company.
 - For example, some recent US issues have come with 0% redemption yield and conversion premiums above 40%, perhaps because of the market’s strong appetite for these credits, because of other generous terms (e.g. investors’ put features), or because the embedded conversion option is valuable (e.g. high stock volatility).
- Market opportunity/timing:
 - Companies may take advantage of a sharp rise in their equity to opportunistically issue a convertible on favourable terms to the issuer when their stock is attracting positive sentiment.
 - Alternatively, cyclical companies may be keen to issue a convertible bond when their stock price is near a trough because selling equity at such low levels is not desirable. With a convertible, the company achieves a low cost of funding until such time as the stock price recovers when the bonds may be converted into equity.
 - They may also launch a well-priced deal following a period of pent-up demand for convertibles in the market, e.g. if there have been few new issues during this period.
- New/broader investor groups:
 - Companies that tap the capital markets often may find that their traditional financing sources, such as straight debt and/or equity investors, have been exhausted.
 - Convertible bonds attract additional investor groups, such as hedge funds, outright convertible funds and institutional trading desks.
 - A broader investor base (a) diversifies the company’s sources of capital and (b) raises their market profile, particularly after a successful convertible issue.
- Capital structure/reporting benefits:

- As hybrid products, convertibles may be accounted for in various ways depending on accounting rules and on its terms and features.
- Convertibles are usually treated as debt on the balance sheet, but their complexity and flexibility may enable the company to structure its finances in terms of debt scheduling and possibly equity dilution. Notes to the accounts should explain key terms of the bonds, e.g. any early redemption features.
- For example, in the US, contingent convertibility may mean that the company need not report earnings per share on a fully diluted basis because the bonds are not considered to be convertible.
- Tax Advantages:
 - Convertibles can also be used to offset or defer tax liabilities.
 - For example, also in the US, contingent payment on an otherwise zero-coupon bond may enable the company to deduct interest as an expense for tax purposes at the rate of its overall cost of debt, giving potentially huge savings (relative to its much smaller real interest payments on the bond).
 - For companies wishing to sell an equity stake that would incur a capital gains tax charge, exchangeable bonds enable that liability to be deferred until such time that the bond is converted into shares. This has been a popular strategy in recent years for large insurers and conglomerates, etc, with cross-holdings.

New Issue Trade-off: Premium Vs Yield

Figure 5: Trade-Off Between Premium Vs Yield for New Issues

Issuers have a trade-off between achieving as low a yield and as high a conversion premium as possible



Source: Barclays Capital.

- When analysing terms for a potential convertible structure, the company usually faces a trade-off between wanting to achieve as high a conversion premium and as low a yield to maturity as possible, for a given issue price, as illustrated in Figure 5.
- Put another way, the company must decide whether to sell a relatively valuable conversion option in return for paying a lower yield (or smaller cash flows), or to sell a less valuable farther out-of-the-money option but having to pay a greater yield (or larger cash flows).

- The decision usually depends on the company's cash flow and management objectives, as well as marketability of the convertible product.

Who Buys Convertibles and Why?

A wide range of investors buys convertible securities for various reasons, as they have become an increasingly popular and better-understood asset class.

Equity funds buy convertibles for downside protection and/or yield advantage

- Equity funds: downside protection and/or yield advantage.
 - Performance is usually dictated by total returns of their selected portfolios of equities relative to benchmarks, e.g. stock market indices.
 - Convertible bonds with low premium or high parity (i.e. in the money) may be purchased instead of equity at little extra cost to provide downside protection if markets fall, thereby outperforming competitors and benchmarks.
 - Convertible bonds may also offer a yield advantage over equity if the coupon yield (or 'running yield') and/or redemption yield exceed the dividend yield on the ordinary stock.

Fixed income funds buy convertibles for equity upside or to diversify their portfolio

- Fixed income and high yield investors: upside participation, credit exposure.
 - Like their equity counterparts, fixed income fund managers aim to deliver superior total returns to their peers and their benchmarks. A bullish view on a company may suggest buying out-of-the-money convertibles instead of straight bonds for their equity upside 'kicker'.
 - Also, some companies only have convertible rather than straight debt outstanding, so investors seeking exposure to those credits may look to the convertible universe to meet their diversification needs in terms of issuer, sector, rating, maturity or yield requirements.
 - Many convertibles trade at yields that are more attractive than their straight debt counterparts. As such, investors have increasingly turned to convertible bonds to generate excess returns.
 - Further down the credit spectrum, 'busted' convertibles (perhaps defined as being far out of the money, or having a high probability of default) may attract 'vulture' funds, recovery funds and other niche players who either bet on a turn-around in the company's fortunes or can extract value from its assets in a default or liquidation scenario.

Outright convertible funds buy 'cheap' bonds on a valuation or risk-return basis

- Convertible funds: valuation and security-specific strategies.
 - These funds typically manage portfolios of various convertible securities on an outright basis.
 - Asset allocation is based on geographic, industry or risk/reward profiles.
 - Security selection is valuation-based, e.g. managers may buy a convertible that is 'theoretically cheap' (market price < theoretical value) believing that its price will rise towards its theoretical value.
 - Theoretical cheapness is relative and not necessarily reliable. Some bonds are cheaper than others and there may be reasons for persistent cheapness that are absent from theoretical models, e.g. corporate activity (see below). 'Cheapness' may be risk-adjusted.
 - Several convertible indices now exist, which funds may use to benchmark their portfolios.

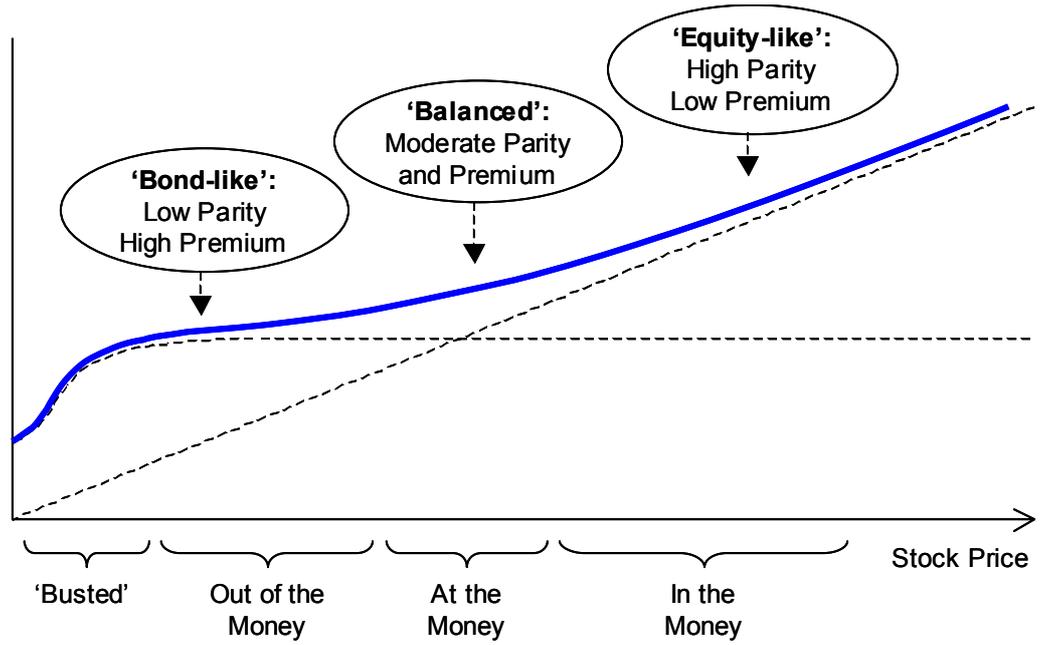
Hedge funds buy convertibles seeking to arbitrage their embedded options

- Hedge funds: leveraged option trading and hedging-based strategies.
 - Given the embedded options in convertibles, it is not surprising that hedge funds have become dominant players in the secondary markets. Their strategies are varied and include the following:
 - Many hedgers buy 'balanced' convertibles, i.e. with a mix of equity and bond characteristics, maintaining a market-neutral position in the underlying equity by hedging the conversion option with a short position in the equity. They then seek to profit from an increase in the implied volatility of the conversion option that they are long, manifested by a 'richening' of the convertible bond itself.
 - They may also profit from the ongoing hedging trades *per se*, i.e. through higher 'realised volatility' of the stock relative to the implied volatility level at which the convertible bond was purchased.
 - Some hedge funds strip out the conversion option on the underlying equity using an 'asset swap', in which the counterparty effectively purchases the fixed income part of the convertible bond, leaving the hedge fund with only the option, resulting in highly geared exposure to the underlying equity. 'Credit default swaps', in which a counterparty offers default protection in return for regular 'insurance' payments, also serve to hedge out the credit risk, but do not affect the gearing.
 - Other hedge funds may simultaneously buy and sell a convertible bond and a 'vanilla' option on the same equity, or a convertible and a straight bond from the same issuer, in order to isolate that part of the convertible that attracts the investor, again leading to gearing.

Risk-arb and specialist funds buy convertibles to exploit M&A and other corporate events

- Risk-arbitrage/specialist funds: corporate activity and special situations.
 - In a takeover or merger situation there may be terms in the offering circular of a convertible bond that present trading opportunities whose profitability depends on the outcome of that situation. For example, some bonds can be redeemed early (put) by holders at par value if there is a change of control (as defined in the prospectus).
 - There are a variety of possible corporate activity scenarios, and many 'risk-arbitrage' traders seek to profit by predicting whether a scenario will arise and taking positions accordingly. An example involving equities would be to buy the stock of the putative target company and sell that of the bidder. Convertible bonds can play an interesting role in such strategies because of their sensitivity to changes in volatility and credit as well as in the equity price.

Figure 6: The Different Characteristics of a Typical Convertible Bond



Source: Barclays Capital.

Key Features and Sensitivities

Conversion

- The higher the conversion ratio, the more value for the holder. This is equivalent to a lower conversion price, or to a lower conversion premium (the percentage by which the conversion price exceeds the stock price).
- Some conversion ratios change with time, e.g. stepping up (or down) as incentives (or disincentives) to hold the bond rather than to convert early.

Calls

Investors want calls at later dates, with higher call prices, redemption yields, or trigger levels

- The issuer's option to redeem the bond early is negative for the holder because it removes the time value of the conversion option, so the longer the issue is non-callable and the higher the early redemption price (or yield) the better.
- If the call is conditional, i.e. the issuer may call the bond only if the shares exceed a certain level, then higher trigger levels give better protection for the holder by preserving more of the time value of the conversion option.

Puts

Investors want puts at earlier dates on more occasions, with higher prices or yields

- The holder's option to redeem early may be exercised if the value of the bond would otherwise be less than the early redemption price. Therefore, more put options, earlier put dates and higher early redemption prices/yields are positive for the holder.

Sensitivities Summary

Figure 7 summarises how the key input parameters affect the value of a typical out-of-the-money, at-the-money and in-the-money convertible.

Figure 7: Typical Convertible Bond Sensitivities

Factor	OTM	ATM	ITM
Stock Price, i.e. equity delta	0	+	++
Stock Volatility, i.e. equity vega ¹	0/+	++	0/+
Stock Dividends	0	+	++
Stock Borrowing Cost	0	+	++
Interest Rates (risk-free), i.e. rho	—	—	0
Interest Rate Volatility, i.e. interest rate vega ¹	0	+	0
Credit Spread (or default probability)	—	—	0
Time to Maturity ²	—	-/+	0/+
Coupon Rate	++	+	0/+ ³
Redemption Price	++	+	0
Conversion Ratio	0	+	++
Time to First Call Date ⁴	0	++	+
Call Trigger Level ⁴	0	++	+
Call Prices ⁴	0	++	+
Time to Puts ⁵	—	—	0
Put Prices ⁵	++	+	0
Exchange Rate (stock currency units per bond currency unit), i.e. FX delta ⁶	0	—	—
Exchange Rate Volatility, i.e. FX vega ¹	0/+	++	0/+

Source: Barclays Capital.

It is difficult to generalise the sensitivities for 'busted' convertibles because they depend on the particular scenario of potential default or restructuring. However, when discussing valuations, we will see (in subsequent sections) that there are some important patterns typically exhibited by far out-of-the-money convertibles.

Notes:

¹ Volatility effects are greatest when the options are approximately at the money, and have less effect for further in or out-of-the-money options.

² A longer time to maturity is generally negative for the bond-like part of a convertible because its yield is usually below the market's risk-free yields in order to pay for the conversion option. The value of this option, by contrast, is greater if it expires later.

³ Far in-the-money convertibles may be worth more than parity if the annual cash flows on the bond exceed those of the stock and the bond should therefore be held rather than converted.

⁴ Issuer calls effectively limit the upside potential and 'time value' in a convertible, so later call start dates, higher trigger levels and/or higher call prices are positive.

⁵ Holder puts effectively offer greater downside protection on specific future dates, so investors prefer as many as possible, sooner rather than later and at high prices.

⁶ For cross-currency bonds, the exchange rate factors into parity in the same way as the stock price. Note, however, that this rate may be quoted inversely.

Traditional Valuation Models

Here, we describe some frameworks for valuing convertibles as used by practitioners for several years. Some, such as the popular blended discount model (implemented in a discrete tree or grid) have been fairly successful. But since the recent market dynamics have emphasised the importance of credit risk for convertibles valuation, some extensions and refinements to this framework have been suggested to better address the credit element – some of those are considered in the subsequent section.

Bond Plus Option / Warrant Model

Convertible Bond Value = Straight Bond Value + Option (Warrant) Value.

'Bond plus Warrant' model is too simple . . .

This is the simplest 'standard' model, but is rarely used now because of its oversimplicity. It is valid only if the convertible can be physically split into a straight bond and options or warrants that can be freely and separately priced and traded. Such separable issues were popular in the 1980s and early 1990s, for example in Japan, where companies frequently issued bonds with warrants attached.

Drawbacks of this valuation model include (depending on implementation):

- Calls and puts, as options on the whole convertible package (rather than on the bond or the option only), may not be handled well.
- Sensitivities may be incorrect, e.g. these models can give equal sensitivity to credit spreads whether parity is high or low, i.e. they assume the credit sensitivity of the straight bond.
- What is the 'strike price' of the option? This depends on the bond's cash flows, which are uncertain (except at maturity).

One-Factor Models: Stock Price

Here we consider some more appropriate valuation models, from first principles. This begins with some standard derivatives theory (as discussed in greater detail in J. C. Hull's book, *Options, Futures, and Other Derivatives*, 4th Edn, Prentice-Hall, 2001).

. . . better to consider a convertible bond as a stock price derivative

As with other derivatives models, convertible bond models make assumptions about the random dynamics of one or more underlying market variables on which its value depends. The most obvious of these variables is the equity into which the holder may convert by surrendering the bond. Stock price is therefore a random variable in any theoretical convertible bond valuation model, and has the following properties:

- The value f of the convertible bond at time t is a function of the then stock price S , i.e.

$$f = f(S(t), t).$$

Assume lognormal distribution for stock

- It is assumed that $S(t)$ follows geometric Brownian motion, or is lognormally distributed, and satisfies the local stochastic equation

$$dS = \mu S dt + \sigma S dz$$

where dS is a small change in the stock price, dt is a small time interval, μ is the expected rate of return on the stock (annualised and continuously compounded), σ is the stock price volatility (annualised) and dz is a random variable that follows a Weiner process (i.e. it is normally distributed with mean zero and variance dt).

Set up a risk-free portfolio of long the bond and short stock

- A portfolio consisting of a long position in the convertible bond and a short position in a number, Δ , of shares (multiplied by the conversion ratio) can be

constructed by choosing $\Delta(t)$ such that the portfolio is instantaneously risk-free with respect to changes in S .

Derive the PDE whose solution is the fair value of the bond, but numerical methods are required to solve it

Ito's Lemma is used to derive the stochastic equation for $f(S, t)$, leading to a partial differential equation (PDE) that is independent of the random variable dz , i.e. that is fully deterministic. The solution of this equation is the convertible bond value.

However, there are no accurate closed-form solutions of this equation for convertible bonds, so it must either be solved by numerical approximation or another framework is required. Tree or lattice models provide one such alternative and are widely used.

Binomial Trees

A common tree model is the binomial, where the stock price can move to two discrete levels over a short time period. The methodology is outlined here.

Building the Stock Price Tree

- Time to maturity T (in years) is divided into a large number N of uniform brief time intervals of length $\delta t = T / N$.
- Over the time interval δt the stock price moves either from S up to $S u$ ($u > 1$) with probability p ($0 < p < 1$) or down to $S d$ ($d < 1$) with probability $1 - p$.
- The principle of risk-neutral valuation shows that in the risk-neutral world, the expected stock price after time δt is $S \exp(r \delta t)$ where r is the continuously compounded risk-free rate over the period δt . This implies

$$p u + (1 - p) d = \exp(r \delta t).$$

- The variance of the proportional change in the stock price after time δt is $\sigma^2 \delta t$, where σ is the (annualised) volatility of the stock price. This gives

$$p u^2 + (1 - p) d^2 - [p u + (1 - p) d]^2 = \sigma^2 \delta t.$$

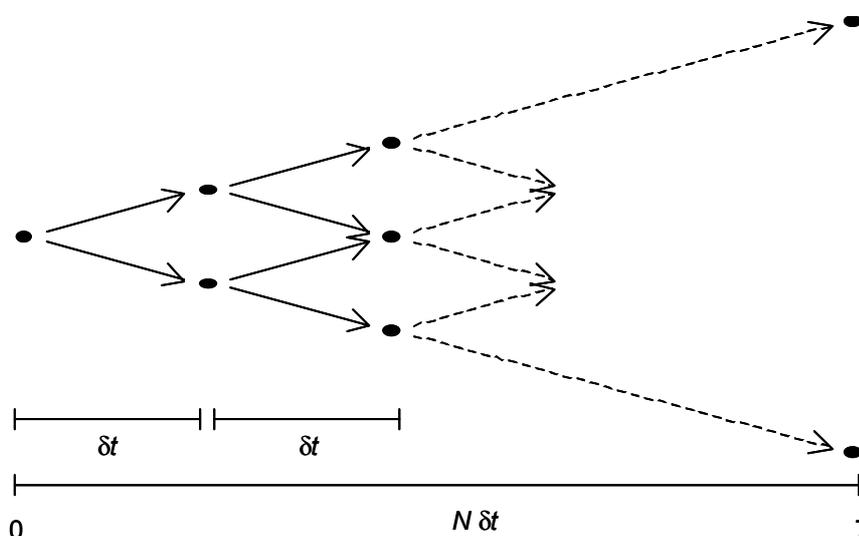
- These two equations in three unknown parameters (u, d, p) leave one degree of freedom to impose an additional relationship. This is often taken to be

$$u = 1 / d$$

so that the branches of the tree recombine at each node (except the uppermost and lowermost nodes), which is computationally efficient, as shown in Figure 8.

Build tree by assuming stock price moves up or down over a time step, with probabilities given by risk-free rate and stock volatility

Figure 8: Binomial Tree of Future Stock Prices



Source: Barclays Capital.

- Solving for the two other parameters (assuming a small δt) gives

$$u = \exp(\sigma \sqrt{\delta t}), d = \exp(-\sigma \sqrt{\delta t}) \text{ and } p = [\exp(r \delta t) - d] / (u - d).$$

Given these tree parameters, future stock prices and risk-neutral probabilities can be derived for all nodes in the tree, both in the time and stock price directions.

Valuing the Convertible Bond in the Tree

Bond is valued by risk-neutral valuation . . .

Let $f_{i,j}$ be the convertible bond value at the i -th time step ($i = 0, \dots, N$) and the j -th stock price level ($j = 0, \dots, i$), i.e. $f_{i,j} = f(S_{i,j}, t_i) = f(S u^j d^{i-j}, i \delta t)$.

The method involves stepping backwards in time through the nodes in the tree and solving recursively for $f_{i,j}$ as follows:

- At maturity (time T), the value $f_{N,j}$ at each terminal node is the greater of parity and the redemption price (possibly including a coupon payment). Denoting the conversion ratio at time t_i by CR_i and redemption amount by Red , we have

$$f_{N,j} = \max\{ CR_N \times S_{N,j}, Red \}.$$

- Prior to maturity, risk-neutral valuation states that the value of the convertible bond is the present value of its (risk-neutral) probability-weighted expected future cash flows, including the discounted value of any coupons between t_i and t_{i+1} (denoted by $Cpns_i$, say). Hence

$$f_{i,j} = \exp(-r \delta t) [p f_{i+1,j+1} + (1-p) f_{i+1,j}] + Cpns_i.$$

- Early conversion by the holder must also be considered (if permissible), so

$$f_{i,j} = \max\{ \exp(-r \delta t) [p f_{i+1,j+1} + (1-p) f_{i+1,j}] + Cpns_i, CR_i \times S_{i,j} \}.$$

- Calls for early redemption must also be considered (if permissible and provided any trigger conditions on the stock price are satisfied). Denoting the call price at time t_i by $Call_i$ and given that the holder may convert if called by the issuer, we now have

$$f_{i,j} = \max\{ \min\{ \exp(-r \delta t) [p f_{i+1,j+1} + (1-p) f_{i+1,j}] + Cpns_i, Call_i \}, CR_i \times S_{i,j} \}.$$

- Finally, puts for early redemption must also be considered (if permissible), so denoting the put price at time t_i by Put_i , the value at each node is

$$f_{i,j} = \max\{ \min\{ \exp(-r \delta t) [p f_{i+1,j+1} + (1-p) f_{i+1,j}] + Cpns_i, Call_i \}, CR_i \times S_{i,j}, Put_i \}.$$

The decision process at each node is therefore equivalent to

$$\text{Convertible bond value} = \max\{ \min\{ \text{hold the bond, issuer calls} \}, \text{holder converts, holder puts} \}.$$

By solving backwards for $f_{i,j}$ in this way, the current convertible bond value is $f_{0,0}$.

Stock Dividends

Future dividends can be modelled as fixed amounts or as yields

If the underlying stock pays dividends then the size and timing of these may be known or accurately forecasted for the next year or so. Beyond that period, different assumptions for future dividends can be handled appropriately in the binomial tree.

- Fixed discrete dividends:
 - All future dividend amounts and ex-dividend dates during the life of the bond are explicitly specified.
 - Build the binomial tree by subtracting from $S_{i,j}$ at each node the present value of all remaining dividends (rather like building the tree in the forward of the stock).

This method is more valid for shorter maturities and lower dividend levels.

- Proportional discrete dividends:
 - All future dividends during the life of the bond are at a specified percentage, the 'dividend yield', of the prevailing stock price.
 - Build the tree by scaling down $S_{i,j}$ at each node following a dividend ex-date by a factor of 1 minus the dividend yield.

This method is more valid for longer maturities and if the future ex-dividend dates can be predicted.

- Proportional continuous dividends:
 - Dividends are modelled as a continuous dividend yield, i.e. paid continuously at a rate proportional to the prevailing stock price.
 - This effectively reduces the rate at which the stock price grows, so the tree is built using risk-free rates of r minus the dividend yield.

This may be better for longer maturities, when the timing of future dividends is unknown, or if (as for some indices) there are many dividends per year.

Stock Borrow Costs

These costs may be incurred in setting up a risk-free portfolio of long bonds and short underlying shares if these shares need to be borrowed from a stock lender.

Borrow costs are usually paid continuously (daily, in practice), as a percentage of the stock price. Stock borrow therefore has the same impact as a continuous dividend yield on the value of this portfolio and is also subtracted from the growth rate r .

Time-Dependent Rates

In reality, risk-free rates depend on time, i.e. $r = r(t)$, as may the dividend rates/yields and stock borrow costs.

This can be implemented using the above formulae and methodology: the risk-neutral probabilities p and $(1 - p)$ also become time-dependent.

Issuer Credit Risk

But the credit risk of the issuer must be taken into account

So far, the model does not account for the credit risk of the corporate issuer, with all cash flows being discounted at the risk-free rate.

- Assuming that investors require an excess return, h (per annum), for bearing the credit risk of a defaultable entity, the present value of a fixed cash flow C due at time T is

$$C \exp\{- [r(T) + h(T)] T\}.$$

Credit spread is larger for greater credit risk

- h is the 'credit spread' or 'discounting spread', and is larger for weaker credits.
- This approach is widely used for valuing straight bonds which pay a stream of fixed cash flows, but may be less applicable for convertibles where some cash flows are fixed but others arise from proceeds of conversion into equity.
- Indeed, the theory of risk-neutral valuation states that equity-derived cash flows should be discounted at the risk-free rate.
- An alternative discounting methodology is needed for hybrid securities.

Blended Discount Models

Blended discounting uses a mixture of risk-free and risky rates to discount cash flows

Several models propose a 'compromise' or 'blended' discounting rate for the cash flows of a convertible bond in a binomial tree. Some common approaches are:

- Full discounting of the straight bond:

- In stepping back through the tree, the value of an equivalent straight bond at the next time step is discounted using the risky rate ($r + h$) and the residual value of the convertible is discounted at the risk-free rate r .
- This 'residual value' does not include proceeds of exercising a put option, because this cash flow is subject to default by the issuer.
- A crucial drawback, as with the bond-plus-option model, is that it attributes the same credit risk to the convertible as to an equivalent straight bond, regardless of whether it is in or out of the money.

- Probability of conversion-weighted discounting:

- Define the quantity, $q_{i,j}$, at each node in the tree as the (risk-neutral) probability that the bond is converted into equity.
- The value of $q_{i,j}$ depends on the decision outcome at the (i,j) -th node: if the bond is converted into stock then $q_{i,j}$ is 1, if it is redeemed then $q_{i,j}$ is 0, or if it is held until the next time step then

$$q_{i,j} = p_j q_{i+1,j+1} + (1 - p_j) q_{i+1,j}.$$

- Denote the risk-free and risky discount factors over a time-step as DF_i and DF'_i respectively, where

$$DF_i = \exp(-r_i \delta t) \text{ and } DF'_i = \exp[-(r_i + h_i) \delta t].$$

- The rolled-back value of the convertible is similar to before, but with the two possible future values discounted using the conversion probability-weighted discount factors. Hence,

$$f_{i,j} = \max\{ \min\{ p_i [q_{i+1,j+1} DF_{i+1} + (1 - q_{i+1,j+1}) DF'_{i+1}] f_{i+1,j+1} + (1 - p_i) [q_{i+1,j} DF_{i+1} + (1 - q_{i+1,j}) DF'_{i+1}] f_{i+1,j} + \text{Cpns}_i, \text{Call}_i \}, \text{CR}_i \times S_{i,j}, \text{Put}_i \}.$$

- $f_{i,j}$ must be computed before $q_{i,j}$ at each node, since the rolled-back value of the convertible must be known before the decision to exercise any call, put or conversion options, which in turn affect $q_{i,j}$.
- Although this model is intuitively appealing, a disadvantage is that the weighting of the risk-free and risky discounting rates depends only on whether the cash flows are derived from the bond or the equity, not on the magnitudes of these cash flows.

- Delta-weighted discounting:

- This is similar to conversion probability-weighted discounting, except delta at a node, $\Delta_{i,j}$, is used to weight the discount factors, so that

$$f_{i,j} = \max\{ \min\{ p_i [\Delta_{i+1,j+1} DF_{i+1} + (1 - \Delta_{i+1,j+1}) DF'_{i+1}] f_{i+1,j+1} + (1 - p_i) [\Delta_{i+1,j} DF_{i+1} + (1 - \Delta_{i+1,j}) DF'_{i+1}] f_{i+1,j} + \text{Cpns}_i, \text{Call}_i \}, \text{CR}_i \times S_{i,j}, \text{Put}_i \}.$$

- Note that delta can be inferred using subsequent nodes in the tree:

$$\Delta_{i,j} = (f_{i+1,j+1} - f_{i+1,j}) / (S_{i+1,j+1} - S_{i+1,j}).$$

- Delta-weighted discounting is also intuitive, but suffers the same important disadvantage. In fact, it is a very similar model.

- Equity and bond cash flow-weighted discounting:

- Here, the value of the convertible at each node is separated into the values of the equity component, $f^{(e)}_{i,j}$ say, and the bond component, $f^{(b)}_{i,j}$ say, such that

$$f_{i,j} = f^{(e)}_{i,j} + f^{(b)}_{i,j}.$$

- The terminal node values are

$$f^{(e)}_{i,j} = \text{CR}_N \times S_{N,j} \text{ if } S_{N,j} > \text{Red} \text{ and } f^{(e)}_{i,j} = 0 \text{ otherwise}$$

Equity and bond cash flows are discounted at risk-free and risky rates respectively

$$f_{ij}^{(b)} = 0 \text{ if } S_{N,j} > \text{Red and } f_{ij}^{(b)} = \text{Red otherwise.}$$

- The rolled-back values at any node prior to maturity are computed using risk-free discount factors for the equity component and risky discount factors for the bond component, i.e.

$$\begin{aligned} f_{ij}^{(e)} &= DF_{i+1} [p_i f_{i+1,j+1}^{(e)} + (1 - p_i) f_{i+1,j}^{(e)}] \\ f_{ij}^{(b)} &= DF'_{i+1} [p_i f_{i+1,j+1}^{(b)} + (1 - p_i) f_{i+1,j}^{(b)}] + \text{Cpns}_i. \end{aligned}$$

- Decisions to call, put or convert are then made with respect to the whole value of the convertible as before, so that

$$f_{i,j} = \max\{ \min\{ f_{i,j}^{(e)} + f_{i,j}^{(b)}, \text{Call}_i \}, \text{CR}_i \times S_{i,j}, \text{Put}_i \}.$$

- If the bond is converted at the (i,j) -th node then $f_{i,j}^{(e)}$ is set to $\text{CR}_i \times S_{i,j}$ and $f_{i,j}^{(b)}$ is set to zero.
- If the bond is redeemed for cash at the (i,j) -th node due to a call or put then $f_{i,j}^{(e)}$ is set to zero and $f_{i,j}^{(b)}$ is set to either Call_i or Put_i .
- Cash flow-weighted discounting is probably the most acceptable blending model because the two separate components of the convertible bond value are discounted using the correct rates.
- This model is also amongst the most widely used.

Full Discount Model

The blended discount models above rely on intuitive appeal in appropriately discounting the risky and risk-free cash flows. An alternative approach involves returning to some first principles of the theoretical valuation model.

- As before, a portfolio is constructed of a long position in the derivative and a short position in a number Δ of shares which is instantaneously risk-free with respect to changes in the share price S . No-arbitrage principles imply that this risk-free portfolio must earn the risk-free interest rate.
- This is valid for a pure stock derivative, but not for a corporate bond because of the risks of losses due to defaults that cannot be hedged using stock.
- The portfolio must therefore earn an excess return over the risk-free rate: this is given by the credit spread, h .
- The bond is valued using the standard binomial methodology as described above but with $r(t)$ replaced by $r(t) + h(t)$.

Full discount model uses risky rates to grow the stock and to discount all cash flows

- The binomial tree parameters therefore become

$$\begin{aligned} u &= \exp(\sigma \sqrt{\delta t}), \quad d = \exp(-\sigma \sqrt{\delta t}), \quad DF'_i = \exp[-(r_i + h_i) \delta t] \text{ and} \\ p_i' &= [(1 / DF'_i) - d] / (u - d). \end{aligned}$$

- Similarly, the discounting formula becomes

$$f_{i,j} = \max\{ \min\{ DF'_{i+1} [p_i' f_{i+1,j+1} + (1 - p_i') f_{i+1,j}] + \text{Cpns}_i, \text{Call}_i \}, \text{CR}_i \times S_{i,j}, \text{Put}_i \}.$$

- Despite its sound theoretical basis, the full discount model is not widely used compared to the blended discount model.
- Some unanswered questions with the full discount approach are:
 - Does the stock hedge truly have no role in hedging instantaneous losses due to default?
 - Is it valid to consider instantaneous default when this is arguably a somewhat continuous process in reality, which could be described by continuous changes in $h(t)$?

Example: France Telecom 4% 2005:

This four-year €3.5bn convertible bond was issued on 21 November 2001, with a coupon of 4% and a conversion price of €72. It is callable at par after two years, subject to a 110% stock price trigger in the third year.

The valuations in Figure 9 to Figure 12 show that there is little difference between the blended discount and full discount models. Even when the credit spread is increased to 500 bp above LIBOR (for illustrative purposes only: at the time of writing, this would be grossly excessive for this issuer), the greatest difference in theoretical values is only approximately 1.5 points and in delta is approximately 3 points. The differences are much less for smaller (and, in this case, more appropriate) spreads.

Figure 9: France Telecom 4% 2005 Value (Credit Spread = 175 bp)

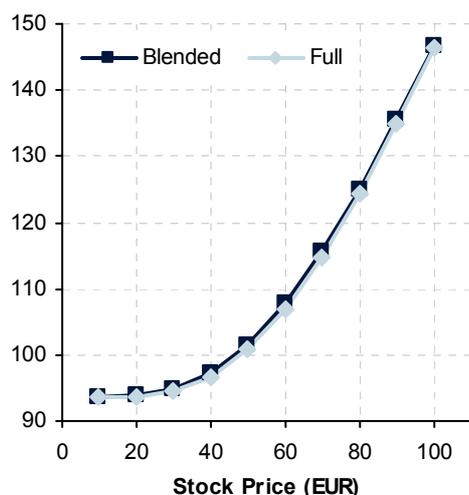


Figure 10: France Telecom 4% 2005 Delta (Credit Spread = 175 bp)

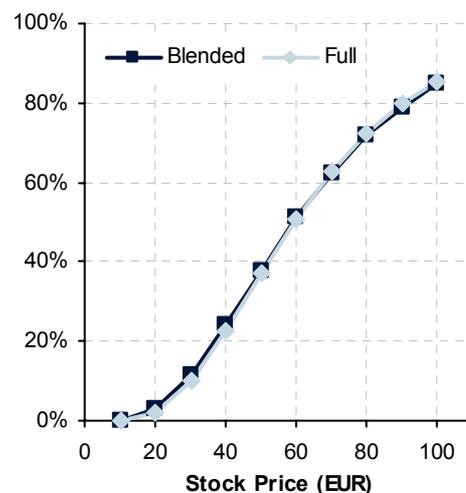


Figure 11: France Telecom 4% 2005 Value (Credit Spread = 500 bp)

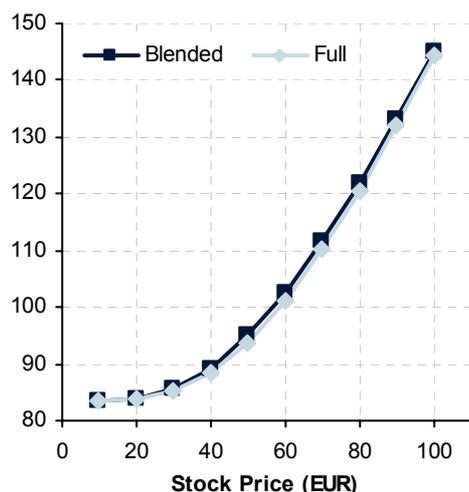
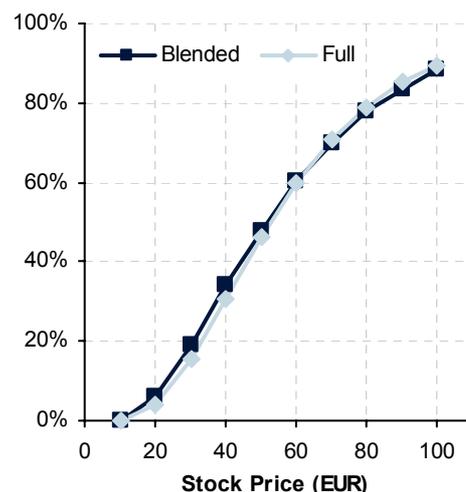


Figure 12: France Telecom 4% 2005 Delta (Credit Spread = 500 bp)



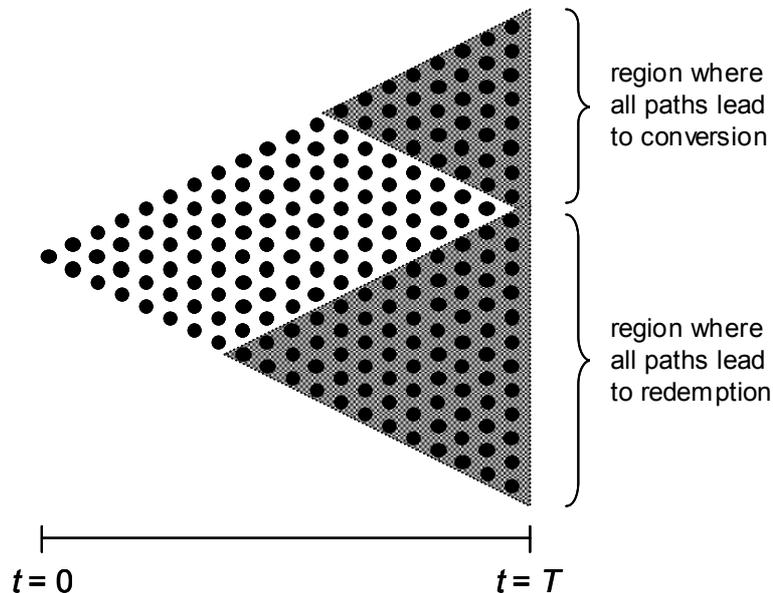
Source: Barclays Capital.

Optimisations

There are certain modifications that can improve either the efficiency or accuracy of the binomial valuation method for convertible bonds, including:

- Omitting any unnecessary calculations in regions of the binomial tree where the outcome is certain, for example:

Figure 13: Binomial Tree Illustrating Regions of Certain Outcomes



Source: Barclays Capital.

- Nodes in the lower part of the tree where all possible paths lead to redemption at maturity (as in Figure 13).
- Nodes in the upper part of the tree where all paths lead to conversion (as in Figure 13).
- Alternatively, nodes in the upper part of the tree where the issuer would call the bond, perhaps 'forcing' the holder to convert.
- For a bond without calls and puts, this simplifies the calculations for nodes within the shaded regions in the diagram above. (Note: similar regions can be deduced when there are calls and puts.)
- Analytic valuation of the 'rolled-back' values at penultimate nodes.
 - At each penultimate node, it may be possible to calculate the value of the conversion option using an analytic formula instead.
 - This leads to more accurate valuations, particularly for 'critical' penultimate nodes, i.e. where parity is close to the redemption price, i.e. where the conversion probability is not close to 0 or 1.
- Placing nodes exactly on the call trigger level (for conditionally callable bonds) may also improve accuracy, by appropriately choosing the tree parameters u , d and p .
- American-style options are not accurately modelled by testing for early exercise at each discrete node because the holder's conversion option and the issuer's call option can be exercised anytime (subject to permissibility). Some techniques are available for improving the accuracy, but given that the conversion option is a type of equity call options (rather than a put) these refinements are probably not very significant.

Binomial Versus Trinomial Trees

Trinomial trees are similar to binomials . . .

- Trinomial trees assume that the stock moves to one of three different levels at the next time step (compared with two levels for binomial trees): up, unchanged or down.
- One advantage of using a trinomial tree is the ability to place a node on the call trigger level at each time step, rather than at every alternate time step in a binomial tree.
- For the same number of time steps, N say, the trinomial tree requires more nodes than the binomial, namely $(N + 1)^2$ instead of $(N + 1)(N + 2)/2$, but the increased number of calculations is generally compensated for by the improved accuracy. Overall, these methods do not generally exhibit a substantial difference in accuracy for a comparable run-time.
- In other respects, the methodology is very similar to the binomial, as are the issues concerning discounting of potentially risky cash flows.

. . . and may need fewer time steps because of their accuracy benefits

Finite Difference Methods

Solving the PDE numerically is an alternative to risk-neutral valuation using a tree

An alternative to risk-neutral valuation using a binomial (or trinomial) tree is numerical solution of the Black-Scholes-Merton partial differential equation for $f(S, t)$:

$$\partial f / \partial t + r S \partial f / \partial S + \frac{1}{2} \sigma^2 S^2 \partial^2 f / \partial S^2 = r f$$

with boundary conditions

$$f(S, T) = \max\{CR \times S, Red\}, \quad f \rightarrow \text{bond floor as } S \rightarrow 0 \quad \text{and} \quad \partial f / \partial S \rightarrow CR \text{ as } S \rightarrow \infty.$$

The method involves solving the above differential equation backwards in time (as with the tree methods) over a discrete, rectangular grid of points in (S, t) -space of separation δS in the stock price direction and δt in the time direction.

Derivative terms in the PDE are approximated by discrete finite difference formulae

The value of f is obtained at each point by approximating the derivative terms in this equation with discrete finite difference formulae.

- 'Central' finite differences are used to approximate the $\partial f / \partial S$ and $\partial^2 f / \partial S^2$ terms, so at the $(i, j)^{\text{th}}$ grid point these terms become

$$\partial f / \partial S_{i,j} \approx (f_{i,j+1} - f_{i,j-1}) / 2 \delta S \quad \text{and} \quad \partial^2 f / \partial S^2_{i,j} \approx (f_{i,j+1} - 2 f_{i,j} + f_{i,j-1}) / (\delta S)^2.$$

- Approximation of the $\partial f / \partial t$ term depends on whether forward or backward finite differences are used, leading to the explicit or implicit scheme, respectively.
- Explicit scheme (forward difference; Figure 14):

$$\partial f / \partial t_{i,j} \approx (f_{i,j} - f_{i-1,j}) / \delta t.$$

- At the $(i, j)^{\text{th}}$ grid point, substituting these finite difference approximations for the derivative terms in the differential equation above gives an expression containing only one unknown, $f_{i-1,j}$.
- It is then straightforward to calculate the value of f at each point at the previous time step recursively back to the present time (applying suitable boundary conditions for grid points next to these boundaries).
- In doing so, we also obtain f not only at the desired stock price level, but also at each stock price level in the discrete grid and can thus obtain a whole valuation 'scenario' or 'curve'.

Explicit scheme is simple to implement but can be unstable

Implicit scheme requires more calculations but is numerically stable

- Implicit scheme (backward difference; Figure 15):

$$\partial f / \partial t_{i,j} \approx (f_{i+1,j} - f_{i,j}) / \delta t.$$

- Now, at the $(i, j)^{\text{th}}$ grid point, substituting the finite differences gives an expression containing three unknown quantities $f_{i,j-1}$, $f_{i,j}$ and $f_{i,j+1}$.
- Therefore, unlike the explicit scheme, it is not so straightforward to calculate f at prior time steps because the whole system of algebraic equations must be solved simultaneously for each stock price level at each time step.
- Fortunately, there are methods of solving these equations (beyond the scope of this article) that recursively give the values of f as required.

Figure 14: Explicit scheme

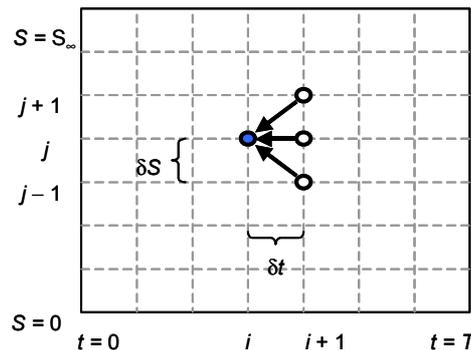
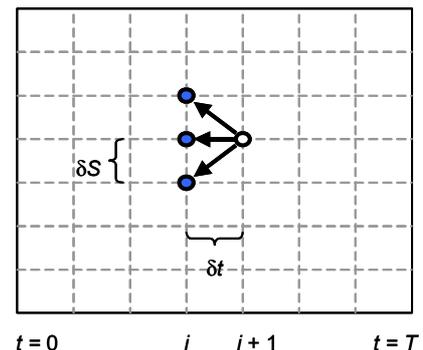


Figure 15: Implicit scheme



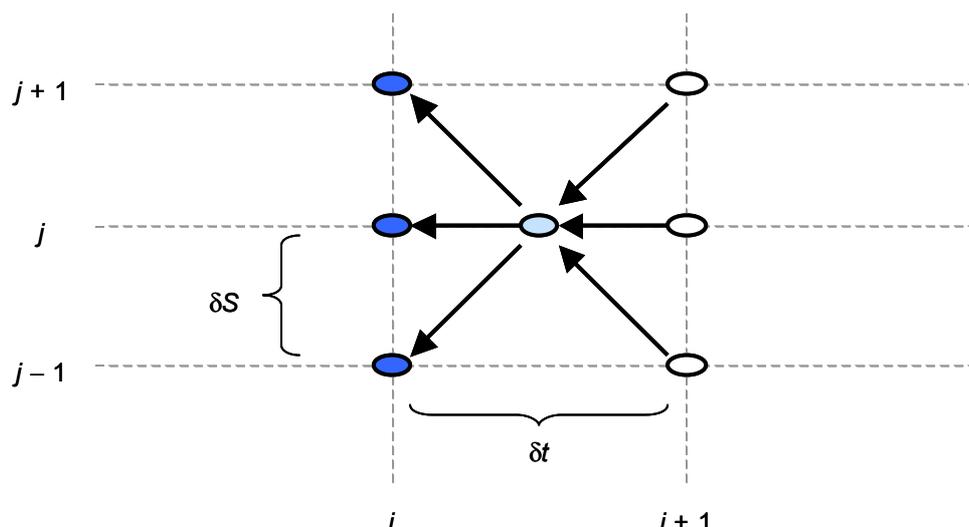
Source: Barclays Capital.

- The explicit scheme is easier to implement than the implicit scheme because the latter requires simultaneously solving a system of algebraic equations.
- But the explicit scheme is unstable unless $\delta t < (\delta S)^2$, so more time steps and hence a longer run-time may be required to ensure stability and accuracy. The implicit scheme, by contrast, is unconditionally stable.
- Choice of finite difference scheme therefore requires a compromise between speed, accuracy, stability, and ease of implementation.
- Semi-implicit scheme (Figure 16):

Semi-implicit scheme is always stable, with improved accuracy

- One widely used and innovative compromise is the 'semi-implicit' finite difference scheme (also known as 'Crank-Nicholson'), which is half explicit and half implicit in time, retaining benefits of both.
- This semi-implicit method splits each time step into two, using a backward difference over one half time step and a forward difference over the other. As such, it effectively uses a central difference approximation in time.
- This method not only has the stability of an implicit scheme, but also has improved accuracy compared to either the equivalent implicit or explicit scheme (on the same discrete grid), because central differences are more accurate than forward or backward differences.

Figure 16: Semi-implicit scheme (detail)



Source: Barclays Capital.

- Convertible bond valuation using finite difference methods:
 - Valuing convertible bonds using such finite difference schemes is more complex than for vanilla options, because of (a) their embedded options and (b) the issuer's credit risk.
 - These embedded options include the holder's conversion and put options, and the issuer's call options. At any point in the grid, these can be tested for exercise (as in the tree methods) by comparing the 'rolled-back' value of f with the proceeds of exercising the relevant options. This is more straightforward for the explicit scheme.
 - One approach towards modelling credit risk relates to the cash flow-weighted blended discount model described earlier for binomial trees. It values the equity and bond cash flow components separately, applying risk-free discount rates for the former and risky rates for the latter, and recombines the two components before testing for exercise of any call, put or conversion options.
 - This approach has the same key advantages (e.g. intuitive appeal, tractability, etc) as the equivalent blended discount model for trees.
 - As with trees, it is unclear whether the finite difference version of the full discount model would be more appropriate than the blended discount model. It is straightforward to implement, however: the risk-free rate r is replaced with the risky rate $r + h$.

PDEs for the equity and bond components of a convertible can be solved using risky and risk-free rates, like 'blended discounting'

Advanced Valuation Models

Problems with Traditional Models

The preceding model framework is generally fine, but has some crucial drawbacks:

Traditional models do not exhibit negative gamma and vega as seen in the market . . .

- a) It does not always generate accurate values and sensitivities, notably for out-of-the-money bonds issued by poor-credit companies. For example, some bonds trade in the market with increasing delta (and therefore negative gamma) and negative vega as parity falls sufficiently, which is not borne out by these traditional models.

The reason for this may be related to another problem:

. . . because credit spreads do not change with stock price

- b) Within the binomial tree, as the stock price moves up and down all other risk factors remain unchanged. This is particularly questionable for the credit spread, h , which for many companies is highly sensitive to large changes in the equity level. Generally, spreads widen as the issuer's equity price falls.

For example, consider the delta from the 'internal finite difference' in a binomial tree:

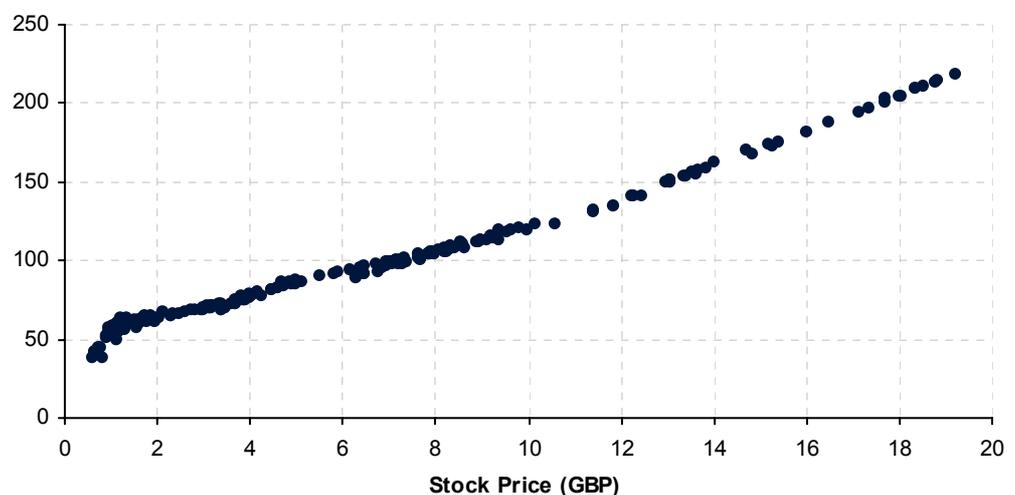
$$\Delta_{0,0} \approx (f_{1,1} - f_{1,0}) / (S_{1,1} - S_{1,0}).$$

This gives delta to be very close to zero when the parity of a convertible is very low, as evidenced by the theoretical profiles for the France Telecom 4% 2005 bond (above).

Credit spreads widen as stock price falls, so the bond floor falls away, with increasing delta, negative gamma

- In reality, delta may increase as the stock price falls, indicating negative gamma, due to widening credit spreads as the risk of default increases. This is demonstrated by the generic valuation profile (see Figure 2) and, for example, by the Colt Telecom 2% 2005 convertible bond:

Figure 17: Colt Telecom 2% 2005 Price (1 January to 1 November 2001)



Source: Barclays Capital.

Figure 17 shows that, during 2001, the price of this convertible fell with the stock price, with decreasing delta (as expected when gamma is positive) until a stock price of approximately £2. Below this level, confidence in this credit appeared to drop sharply, manifested in the bond price falling on a greater delta, indicating negative gamma.

- An alternative is to use external finite differences for delta. The bond is valued once at the stock price S with credit spread h , and again at a lower stock price $S - \epsilon_S$ with a higher credit spread $h + \epsilon_h$ (with ϵ_S and ϵ_h small), and then taking the finite difference

$$\Delta(S; h) \approx [f(S; h) - f(S - \varepsilon_S; h + \varepsilon_h)] / [S - (S - \varepsilon_S)] = [f(S; h) - f(S - \varepsilon_S; h + \varepsilon_h)] / \varepsilon_S.$$

- However, this is inconsistent with the binomial methodology because a downward move in the stock price over a time step would not cause any change the credit spread.

Quasi-Two-Factor Models: Stock Price-Dependent Credit Spreads

Dynamic credit spread depends explicitly on stock price

- As noted, the traditional model performs poorly for volatile, high-yielding, low-parity convertibles. It tends to over-estimate gamma and vega and tends to under-estimate delta. This is essentially because the credit spread does not vary with stock price in the tree.
- A natural extension is to consider dynamic credit spreads $h(t)$ that change in the tree as the stock price $S(t)$ changes, in a manner specified by some explicit relationship, i.e. $h(t) = h(S(t), t)$.
- Therefore, the spread $h(S)$ is only indirectly stochastic, via its dependence on the stock price, so this is described as a 'quasi-two factor' model.
- The functional form for $h(S)$ is assumed to have the following properties.

- The spread becomes infinite as stock price falls to zero because the company defaults (note that this can be modified to account for the recovery value, if any):

$$h \rightarrow \infty \text{ as } S \rightarrow 0.$$

- The spread is a monotonic decreasing function of stock price because a rise in the equity capitalisation should, all else being equal, lead to an improvement in the credit strength of the company due to improved asset coverage and lower financial gearing:

$$dh/dS \leq 0 \text{ for all } S > 0.$$

- As the stock price becomes very large, the spread tends to some non-negative limit h_∞ (say), which reflects the minimum credit risk premium for debt of this ranking and subordination from this issuer:

$$h \rightarrow h_\infty \text{ as } S \rightarrow \infty, \text{ or alternatively, } dh/dS \rightarrow 0 \text{ as } S \rightarrow \infty.$$

- The current spread h_0 and stock price S_0 are consistently calibrated:

$$h(S_0) = h_0 \text{ where } h_0 \geq h_\infty.$$

- A simple relationship that satisfies these properties and contains just one new parameter is

$$h(S) = h_\infty + (h_0 - h_\infty) (S / S_0)^{-k}, \text{ where } k \geq 0.$$

Credit spread – stock price relationship involves one key sensitivity, the k factor

- The decay parameter k measures the sensitivity of the dependence of the credit spread h on the underlying stock price S . As such, this 'k factor' is a very useful and important parameter. By setting $k = 0$ we retrieve the original, traditional model.

Estimation of the k Factor

For any convertible bond, there are several possible approaches towards estimating the value of this parameter by calibration.

k can be estimated theoretically from a firm value model . . .

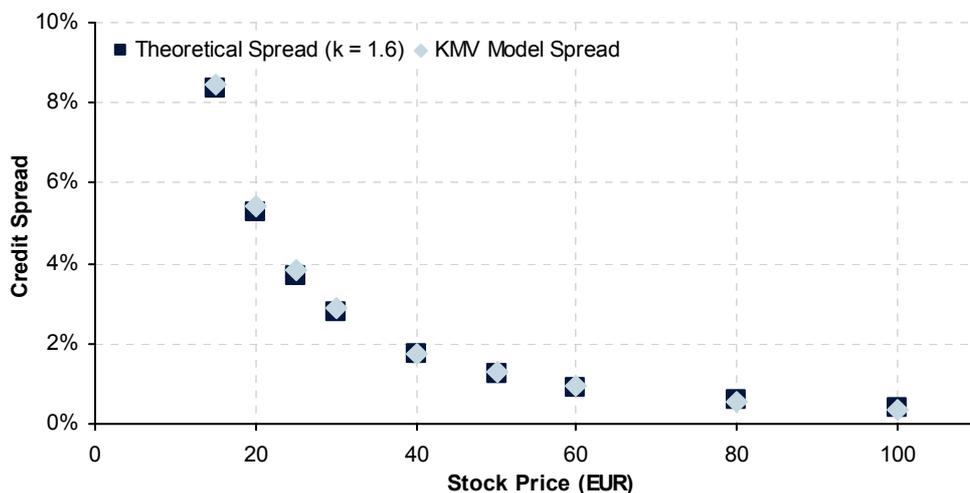
- Using a Merton-type option theory model, such as that provided by KMV Corp, to compute a company's probability of default as a function of its equity price.

This model values the company's equity as a call option on its asset value, struck at some 'default point' that is related to its level of debt. The default

probability then depends on the number of standard deviations from the implied asset value (given the equity value as the input) to this default point.

By computing the default probability for a range of different equity values, a relationship between credit spread and stock price can be obtained. This is qualitatively similar to the functional form for $h(S)$ above, and provides a theoretical means of calibrating the k factor.

Figure 18: Calibration of k for France Telecom using KMV's Model



Source: Barclays Capital and KMV.

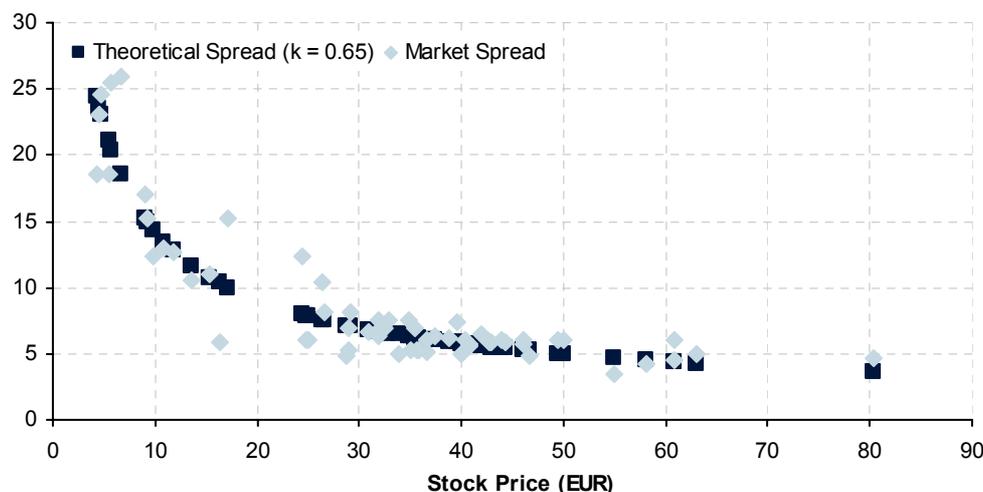
Figure 18 shows that, for France Telecom, our functional form closely fits KMV's model for the relationship between credit spreads and equity prices with a suitable choice of k (here, $k = 1.6$).

... or empirically from straight debt prices

- Empirically, using historic prices/yields of straight bonds from the same issuer with similar ranking, subordination and duration.

Where this is possible and reliable market data exists, we find that the observed relationship between credit spreads (implied from bond prices/yields) and stock prices closely resembles our functional form for some issuers, particularly in the high yield sector.

Figure 19: Calibration of k for Versatel using Market Spreads



Source: Barclays Capital.

Figure 19 shows that, for Versatel, our functional form is a good approximation to the market-observed relationship between credit spreads and equity prices.

Valuation Profiles with $k > 0$

The impacts of equity price-dependent credit spreads can be profound. These include a 'soft' bond floor, in that it decreases when the stock price falls sufficiently, as well as higher delta and lower or even negative gamma and vega as the stock price falls sufficiently, particularly for weaker credits.

Figure 20: Theoretical Value of France Telecom 4% 2005 Convertible Bond

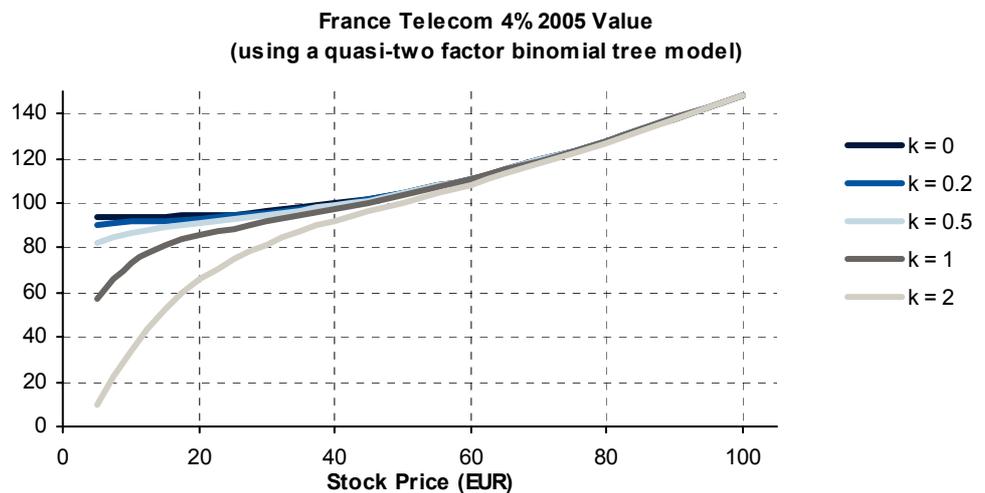
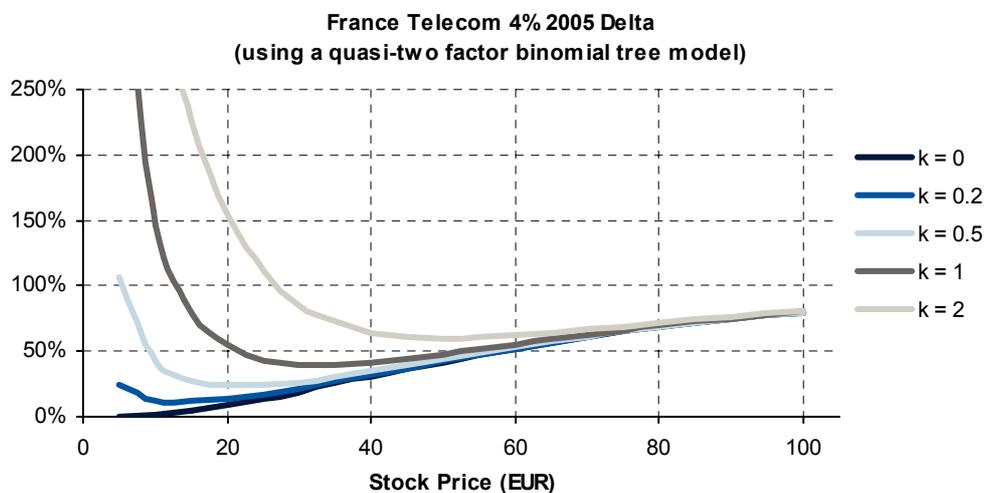


Figure 21: Theoretical Delta of France Telecom 4% 2005 Convertible Bond



Source: Barclays Capital.

Higher k results in a falling bond floor, increasing delta and negative gamma as stock price falls . . .

Figure 20 and Figure 21 show how the effects of the k factor become more exaggerated as k increases (implemented in a blended discount model).

The k factor, like the volatility σ and current credit spread h_0 , is a parameter that must be estimated or implied from the market price and delta, i.e.

$$f(\sigma; h_0; k) = f_{\text{market}} \quad \text{and} \quad \Delta(\sigma; h_0; k) = \Delta_{\text{market}}.$$

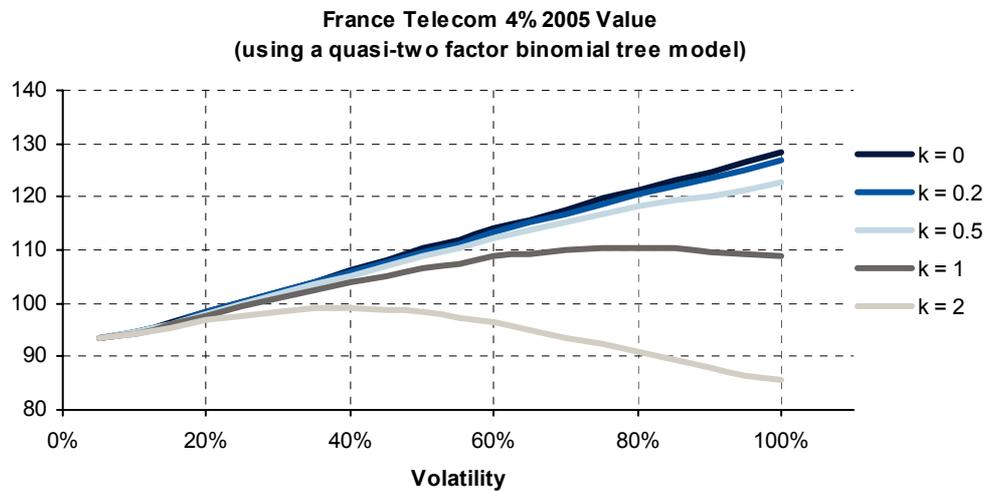
This extra flexibility in parameter determination means that these implied parameter values should be more realistic than with the constraint $k = 0$ in the traditional model.

Volatility Effects with $k > 0$

... and negative vega

For relatively high levels of k , increased volatility may have a negative impact on theoretical values.

Figure 22: Theoretical Value of France Telecom 4% 2005 Convertible Bond



Source: Barclays Capital.

Figure 22 shows how vega decreases as k increases, and may even become negative (here parity is 75).

This can be explained in terms of upside versus downside risk. With out-of-the-money (but not 'distressed') convertibles, a small upward or downward movement in parity may be fairly equivocal. Larger moves may be less so, however, if the credit deterioration on the downside has a greater negative impact than the positive effects of improved credit strength and (possibly) increased equity exposure on the upside.

- This highlights a counter-intuitive convertibles investment strategy:

Traditionally, decreasing volatility is considered to be detrimental to hedgers who seek to profit from volatility through delta hedging. But for sufficiently volatile stocks, bonds with a higher k would outperform those with a lower k in a falling volatility market, and may even gain in value in absolute terms.

- It may also explain why convertible bonds have lower implied volatilities:

Market participants have traditionally observed lower implied volatilities for convertible bonds than would be expected based on (a) historic volatilities and (b) levels implied by other derivatives on the stock (e.g. listed options).

Consider Figure 22, for example: if the market price of the convertible is 105 (say) then the implied volatility in the traditional $k = 0$ model is approximately 35% but if $k = 1$ (say) then it would be closer to 45%.

Convertible investors have also anecdotally stated that, for some bonds, they would never pay above a certain level in implied volatility. A possible explanation for this is provided by the above graph: for large k , theoretical value becomes bounded above as volatility increases, which in the traditional model framework translates to a maximum level for the volatility input. For example, if $k = 1$ then theoretical value cannot exceed approximately 110, which in the ' $k = 0$ ' model means implied volatility cannot exceed 50% (see Figure 22).

This may also explain why implied volatility appears to be bounded above for some bonds

Dynamic credit spreads can be included in blended discount models

Implementation in a Blended Discount Model

In the cash flow-weighted blended discount model, the equity-derived and bond-derived components of the convertible's value are discounted separately at the risk-free and risky rates, respectively, to obtain the 'rolled-back' value at each node.

For example, in a binomial tree the equity component $f_{ij}^{(e)}$ remains unchanged but the bond component becomes

$$f_{ij}^{(b)} = \exp\{-[r_i + h(S;u)] \delta t\} p_i f_{i+1,j+1}^{(b)} + \exp\{-[r_i + h(S;d)] \delta t\} (1 - p_i) f_{i+1,j}^{(b)} + C\text{pns}_i.$$

In this way, the equity-dependent credit spread framework can be implemented in a blended discount or similar model.

Convertible and Exchangeable Bonds

The above theory generally applies only for convertibles. Exchangeables do not usually exhibit a strong relationship between the issuer's credit and the underlying stock price because the shares are of a different corporate entity.

Exceptions to this distinction could occur (a) if the issuer holds a large stake in the underlying company, e.g. Mosel – ProMos 1% 2005, or (b) if the issuing and underlying companies are highly correlated, e.g. SAI – Generali 1% 2004 (both companies are leading Italian insurers).

Hedging Strategies for the k Factor

Investors can hedge k factor using a heavier delta, put options or credit protection

Convertible bond investors have various ways of managing the market effects caused by this relationship between credit spreads and underlying equity prices, including:

- Selling more underlying shares than suggested by a traditional model to hedge the higher delta.
- Buying put options on the underlying shares to hedge both the higher delta and the negative gamma.
- Using credit protection mechanisms such as asset swaps or default swaps to hedge against a widening in the credit spread of the bond.

These strategies need to be implemented more aggressively for higher values of k .

Two-Factor Models: Stock Price and Credit Spreads

Alternatively, model credit spread as a second, correlated random variable . . .

- The stock price-dependent credit spread model is intuitively sensible and appears to provide realistic valuations, sensitivities and implied parameters, but it does constrain the credit spread to have an explicit relationship with the stock price via the functional form $h(S)$.
- In reality, credit spreads contain at least some element of randomness in their own right. This suggests developing a two-factor model in which both stock prices and credit spreads follow separate but correlated random processes.
- However, there are also issues with such a two-factor model, including the extent to which the value of the resulting credit option can be realised in practice and whether the credit risk can be instantaneously hedged.
- Further investigation of this framework compared to the quasi-two factor model is required, and is beyond the scope of this report.

. . . but unclear if this approach is consistent with practical issues in hedging credit risk

Interest Rates and Exchange Rates

Can also model FX rates and interest rates as random variables

Some convertible valuation models provide the option to include interest rates or currency rates as an additional stochastic variable.

-
- Stochastic currency rates may make little difference because its variability can be incorporated in the underlying stock variability, an approach which is generally correct to first order.
 - Stochastic interest rates tend also to have only small impacts, depending on the correlations and on the nature of any embedded options.

Firm Valuation Models

Some convertible valuation models use the asset value of a company as the underlying, stochastic variable rather than the company's equity price. This follows an approach developed by Merton and others in which the equity of a firm is viewed as a call option on its assets. In this framework, a convertible bond initially forms part of the company's liabilities as a debt security but becomes equity if the bond is converted. This depends on the asset value: broadly, if the asset value increases then conversion becomes more likely.

Further discussion of these model variants is beyond the scope of this report.

Summary

A convertible bond provides investors with an option to convert the bond into the issuer's equity, whereas an exchangeable bond is convertible into equity of a different entity. Convertibles may be called by the issuer (usually anytime after the first few years) and may be put back to the issuer by the holder (usually at specific times during the life of the bond). All terms and features are detailed in the prospectus.

At maturity, investors receive redemption proceeds unless parity (the underlying share value of the bond) is higher, in which case they will convert into stock. Before maturity, the holder's conversion option has some time value, which is the convertible's extra value above that of either the equivalent straight bond value or parity. This value is affected by the same market factors as for call options, such as stock volatility, future dividends, borrow costs, etc, and by any structural features such as calls or puts.

Convertibles have become a prominent asset class as issuance has surged in line with investor demand. Companies issue convertible bonds for their relatively low cost of financing compared to straight debt or equity, for their favourable pricing and for their appeal to a wide range of investors. Exchangeable bonds additionally provide a cost-effective means of selling cross-holdings of equity. Investors buy convertibles for their equity upside relative to straight debt, for their downside protection relative to equity, for credit diversification and for the realisable value of their embedded options.

Accurate valuation and risk management of convertibles is complex due to the large number of market factors that affect value. The underlying equity price is a main driver of value, and many models use standard options theory to describe how a convertible's value depends on the stock price, which itself is modelled as a stochastic process. For convertibles, accounting for the issuer's credit worthiness when discounting its future cash flows is crucial. One popular approach, blended discounting, uses risk-free rates to discount equity-derived cash flows and risky rates (i.e. including the issuer's credit spread) to discount bond-derived cash flows.

Traditional models, however, fail to explain why many convertibles exhibit higher delta (i.e. greater equity sensitivity) and possibly negative gamma and vega at low stock prices. This is because they do not recognise the tendency of an issuer's credit spread to widen as its equity value falls. A possible remedy is to model this relationship explicitly, and we have suggested a functional form that closely matches both theoretical firm valuation models and empirical data. This relationship involves a parameter, the 'k factor', which measures the sensitivity of the credit spread to changes in the stock price. When k increases from zero, this model exhibits a bond floor that falls away, increasing delta and lower or negative gamma and vega as the stock price decreases, as observed in reality. This model also explains why convertibles may trade on lower implied volatilities than similar listed stock options and why convertibles with higher k factors may outperform when volatility falls. This dynamic credit-equity relationship can be implemented within a blended-discount model framework, leading to a 'quasi two factor' model in that the credit spread is variable because it depends on the stochastic stock price.

Alternative valuation models can be devised, including two factor models in which both the stock price and credit spread are separate but negatively-correlated stochastic variables. This is arguably a more flexible specification for credit spreads and stock prices, but is more complex and it is unclear whether the credit option embedded in the convertible can be hedged in practice. Other two factor models assume either stochastic interest rates or currency rates. Another possible avenue is a firm valuation model wherein the convertible is treated as debt that may become equity in the future.

Finally, we advise that correct use of a model is as important as its formulation. The values of key inputs such as volatility, credit spread, k , future dividends, borrow, etc, must be estimated appropriately and consistently, given the assumptions of the model.

Appendix

BARCLAYS

29 November 2001

EXCHANGEABLE NOTES SALES PRICING TERMS

Bloomberg
NOVN VX Equity



Reuters
NOVZn.VX

Eur 2.8 billion Dual Tranche Deutsche Bank / Novartis Exchangeable Bonds

FOR INTERNAL DISTRIBUTION ONLY (No Longer Applicable)

OFFERING SUMMARY

Description of Offering:	<ul style="list-style-type: none"> ■ Issuer: Deutsche Bank Finance NV ■ Guarantor: Deutsche Bank AG ("Deutsche Bank") ■ Form of security: Bonds exchangeable into ordinary Novartis common shares ■ Size 2011 tranche: Eur 1,400m offering ■ Size 2010 tranche: Eur 1,400m offering ■ Total size: Eur 2,800m equivalent offering ■ Status: Senior, unsubordinated and unsecured ■ Denomination / Form: Eur10,000 / dematerialised book entry ■ Maturity 2011 tranche: 10 years (6 December 2011) ■ Maturity 2010 tranche: 9 years (6 December 2010) ■ Issue price: 100% of par – both tranches ■ Coupon: 0.00% both tranches ■ YTM – 2011 tranche: 2.75% (annual) ■ YTM – 2010 tranche: 3.125% (annual) ■ Redemption price–2011 tranche: 131.2% of par ■ Redemption price–2010 tranche: 131.9% of par ■ Exchange premium: 28.2% premium over reference local share price at pricing (both tranches) ■ Pricing: One-day bookbuilding and pricing ■ Rating: Bonds will be rated by S&P, Deutsche Bank currently AA (stable) ■ Put feature – 2011 tranche: Puttable in years 3, 5 and 7 ■ Put feature – 2010 tranche: Puttable in years 4 and 6 ■ Call feature (both tranches): Not callable for 3 years, thereafter callable at the accreted principle amount subject to the Novartis shares trading at 130% of the accreted principal amount Hard callable from year 5 ■ Use of proceeds: General corporate purposes ■ Other provisions: Issuer's cash-out option, takeover protection (at bond holders option put at 105% or substitution into new entity), anti-dilution protection, extraordinary dividend protection (3% threshold), no tax gross up, tax, legal, accounting and regulatory call at greater of market value and accreted principal amount, clean up call (25%) ■ US restrictions: No Rule 144a offering; Distribution by Regulation S only ■ Governing law: German law ■ Listing: Luxembourg Stock Exchange (expected) – allocations subject to closing ■ Settlement / date: Euroclear, Clearstream / expected 6 December 2001 ■ Lock-up period: 30 days from settlement
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TIMETABLE

<u>Date</u>	<u>Events</u>	<u>Zurich</u>	<u>London</u>
29 November	Books open		6:30 AM
	Latest books may close		5:30PM
6 Dec	Closing, settlement		

Barclays Capital

European Credit Research

Barclays Capital
5 The North Colonnade
London E14 4BB

Phone: +44 (0)20 7773 9022
Fax: +44 (0)20 7773 7389

Credit Strategy

Gary Jenkins

Global Head of Investment Grade Credit
Research
+44 (0)20 7773 9691
gary.jenkins@barcap.com

Alok Basu

+44 (0)20 7773 8559
alok.basu@barcap.com

Jim Reid

+44 (0)20 7773 9533
jim.reid@barcap.com

Corporates

Juan Carrion (Autos / Media / Leisure)

+44 (0)20 7773 8476
juan.carrion@barcap.com

Suzanne Cassidy (Retail / Chemicals)

+44 (0)20 7773 8482
suzanne.cassidy@barcap.com

Janice Davidson (Tobacco / Transportation)

+44 (0)20 7773 9692
janice.davidson@barcap.com

John Dockery (Industrials)

+44 (0)20 7773 9369
john.dockery@barcap.com

Jon Scoffin (Industrials / Property)

+44 (0)20 7773 9139
jon.scoffin@barcap.com

Michelle Boath (Junior Analyst)

+44 (0)20 7773 8256
michelle.boath@barcap.com

Telecommunication / OEMs

Stephan Michel

+44 (0)20 7773 8392
stephan.michel@barcap.com

Laura Winchester

+44 (0)20 7773 9688
laura.winchester@barcap.com

Milena Ianeva (Junior Analyst)

+44 (0)20 7773 8536
milena.ianeva@barcap.com

Utilities, Oil & Gas

Neil Beddall

+44 (0)20 7773 9879
neil.beddall@barcap.com

Andrew Moulder

+44 (0)20 7773 8558
andrew.moulder@barcap.com

Paula Dahlen (Junior Analyst)

+44 (0)20 7773 8589
paula.dahlen@barcap.com

Financial Institutions

Roberto Bella

+44 (0)20 7773 8418
roberto.bella@barcap.com

Larissa Knepper

+44 (0)20 7773 9138
larissa.knepper@barcap.com

Guido Versondert

+44 (0)20 7773 9799
guido.versondert@barcap.com

Public Sector

George Johnston

+44 (0)20 7773 9690
george.johnston@barcap.com

Research Assistants

Nick Burns

+44 (0)20 7773 8577
nick.burns@barcap.com

Robert Emes

+44 (0)20 7773 8537
robert.emes@barcap.com

Eugene Regis

+44 (0)20 7773 9169
eugene.regis@barcap.com

Convertibles

Douglas Decker

+44 (0)20 7773 8302
douglas.decker@barcap.com

Haidje Rustau

+44 (0)20 7773 8301
haidje.rustau@barcap.com

Judy Ho

+44 (0)20 7773 9682
judy.ho@barcap.com

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